A model including compressible firn applied to crater glacier flow simulation

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Introduction

Shear fluidity as well as compressibility of firn show a strong dependency on the ice volume fraction. We present the following model:

- solution of full compressible Stokes-system with thermo-mechanical coupling (material properties dependent on temperature)
- limited solution of temperature field with respect to negative values of the homologous temperature
- introduction of firn-properties [1] into dynamics

The model is applied to the Gorshkov crater glacier at Ushkovsky volcano, Kamchatka [2]. Simulation results assuming a steady state of the glacier's dynamic/thermodynamic properties are presented.

Governing equations

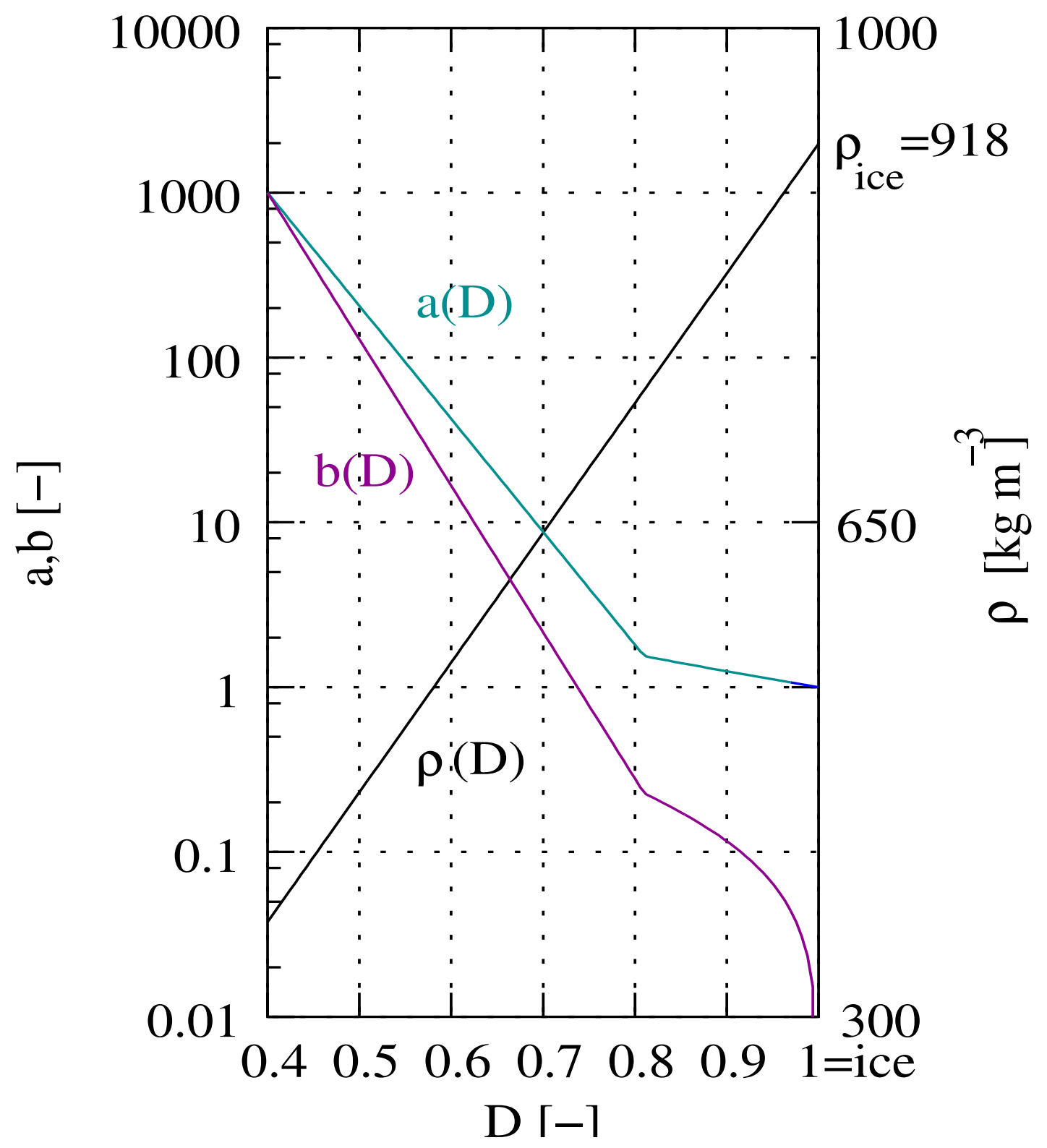
Volume balance: $\dot{\epsilon}_{ii} + \kappa_{\text{cp}}\, p = 0$

Stokes equation: $-p_{,i} + s_{ij,j} = -\rho_{\text{ice}}\, D\, g_i$

Heat Transfer: $\rho_{\text{ice}}\, D\, c \cdot \left(\dfrac{\partial T}{\partial t} + v_i\, T_{,i} \right) = (\kappa\, T_{,i})_{,j}$

Dating (age of ice): $\dfrac{\partial \mathcal{A}}{\partial t} + v_i\, \mathcal{A}_{,i} = 1$

Constitutive relations

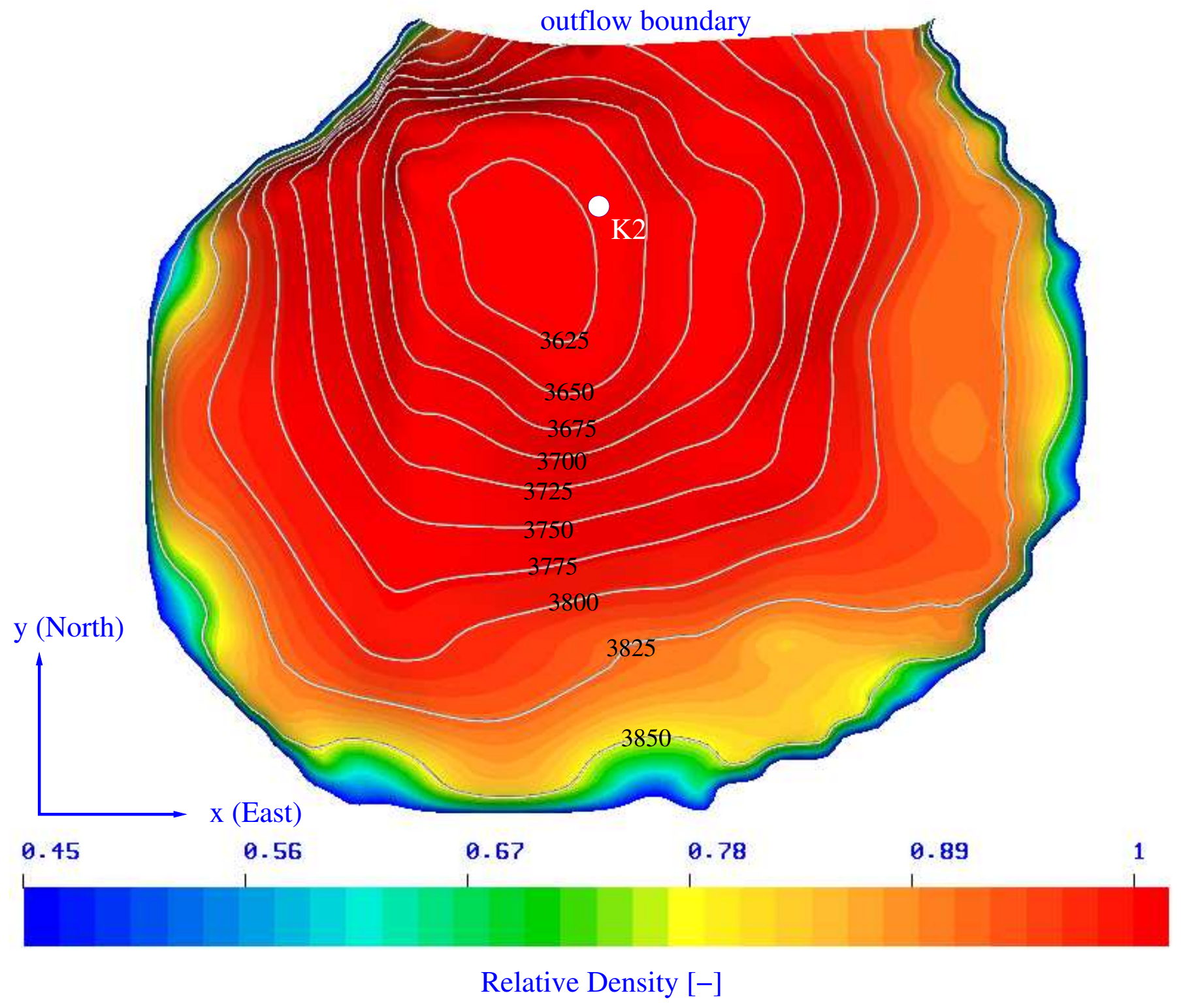


$$s_{ij} = 2\,\eta \cdot (\dot{\epsilon}_{ij} - \delta_{ij}\, \dot{\epsilon}_{kk}/3)$$
$$\dot{\epsilon}_{ij} = 1/2 \cdot (v_{i,j} + v_{j,i})$$
$$\dot{\epsilon}_{\text{D}}^2 = \frac{2\dot{\epsilon}_{ij}^2}{a} + \frac{\dot{\epsilon}_{ii}^2}{b}$$
$$\eta = \frac{1}{a}\,(2E\, A(T'))^{-1/n}\; \dot{\epsilon}_{\text{D}}^{(1-n)/n}$$
$$A(T') = A_0\, e^{-Q/RT'}$$
$$\kappa_{\text{cp}} = b \cdot (a\,\eta)^{-1}$$
$$\kappa = (c_1 - c_2\, D + c_3\, D^2) \cdot \tilde{c}_1\, e^{-\tilde{c}_2\, T[\text{K}]}$$

Nomenclature: v_i…i-th velocity component, T…temperature, p…dynamic pressure, $D = \rho/\rho_{\text{ice}} \in [0,1]$…relative density, $\mathcal{A}$…age of ice, s_{ij}…deviatoric stress tensor component, $\dot{\epsilon}_{ij}$…strain-rate tensor component, η…viscosity, $A(T')$…Arrhenius factor, E…enhancement factor, c…heat capacity, κ…heat conductivity, g_i…i-th component of acceleration due to gravity, $T' = T - T^{(\text{pm})}$…homologous temperature, $T^{(\text{pm})} = T_0 - \beta \cdot p$…pressure melting point, Q…activation energy, R…universal gas constant, β…Clausius-Clapeyron coefficient, L…latent heat, $j_\perp = 570\,\text{kgm}^{-2}\text{a}^{-1}$…averaged accumulation mass flux, j_{m}…reference basal melting rate (obtained from intitial simulation run)

Operating conditions

Imprinted relative density distribution as a function of the flow depth, d (from bore-hole K2): $D = 1 - 0.55 \cdot e^{-0.038\, d\,[\text{m}]}$



The bedrock topography of the Gorshkov crater. The white iso-lines show levels of constant vertical coordinate, z in meters a.s.l., with an offset of $\Delta z = 25\,\text{m}$. The color texture applied on the bedrock surface shows the local values of the imprinted relative density, D. The white dot indicates the location of the bore-hole K2

Boundary conditions

free surface: $T = 256.56\,\text{K}\,(-16.6°\text{C}),\; s_{ij}\, n_j = 0,\; \mathcal{A} = 0$

outflow: $T_{,i}\, n_i = 0,\; v_i\, n_i = \left(\int\limits_{A_{\text{surf}}} j_\perp\, dA - \int\limits_{A_{\text{bed}}} j_{\text{m}}\, dA \right) \Big/ \int\limits_{A_{\text{out}}} \rho\, dA$

bedrock: $j_{\text{m}} = \rho_{\text{ice}}\, D\, v_i\, n_i|_{T'=0} = \dfrac{q_{\text{geo}} - \kappa T_{,i}^{(\text{pm})}\, n_i}{L},\; v_i|_{T'<0} = 0$

$$\kappa\, T_{,i}\, n_i|_{T'<0} = q_{\text{geo}},\; T|_{T'=0} = T^{(\text{pm})}$$

Numerical methods

Implementation using open source FE package Elmer [6]:

- Stabilized Method [3] for advective-diffusive systems (flow, heat transfer)
- "Contact" problem [4] for heat transfer with $T \leq T^{(\text{pm})}$
- Discontinuous Galerkin Method [5] for advection-reaction type systems (dating, evolution of relative density)

Parameter variation

Variation of heat flux at bedrock

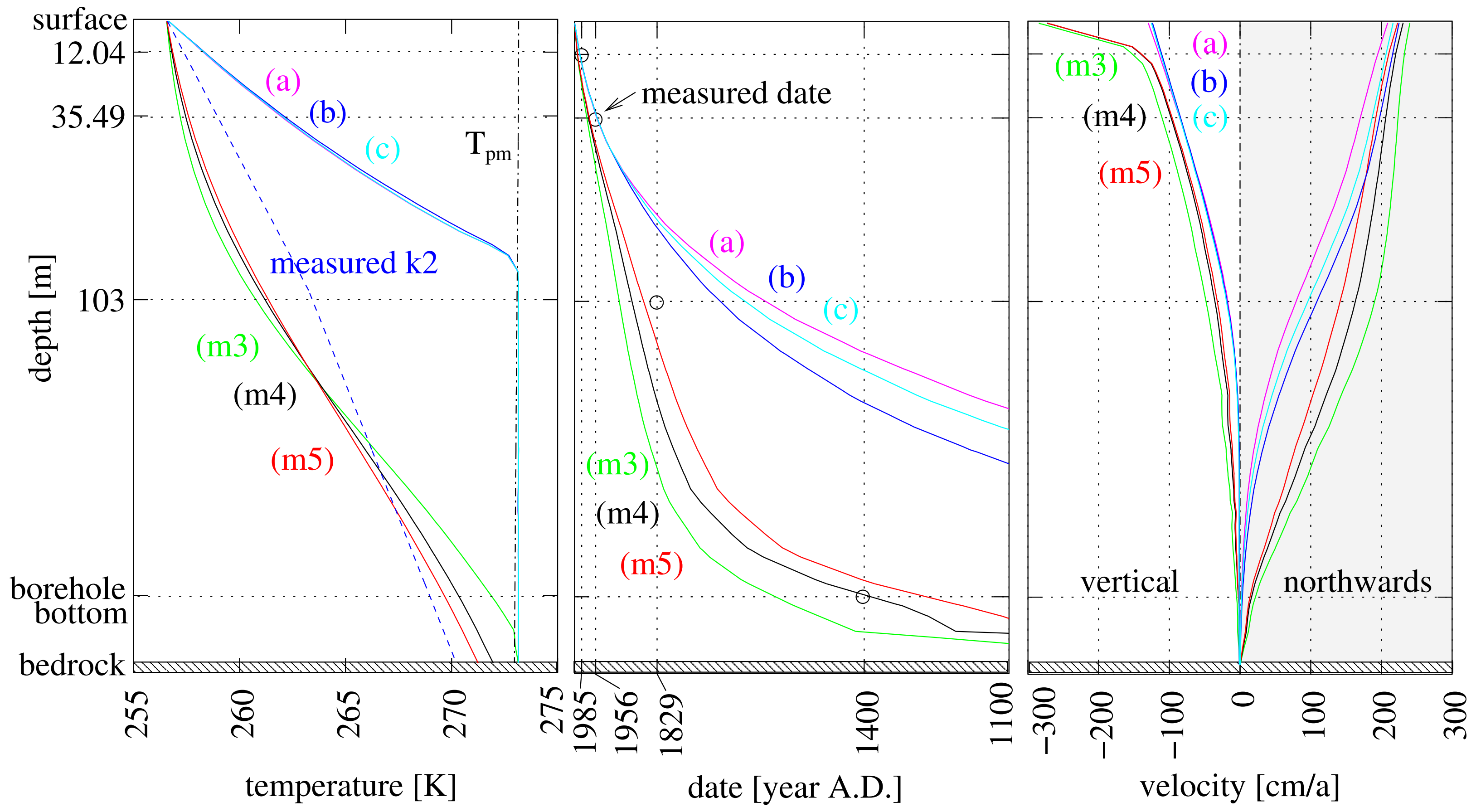
$$q_{\text{geo}} = \underbrace{q_{\text{min}}}_{=0.12\,\text{W}\,\text{m}^{-2}} + \Big(\underbrace{q_{\text{max}}}_{=10\,\text{W}\,\text{m}^{-2}} - q_{\text{min}} \Big) \cdot \left(\frac{z - z_{\text{min}}}{z_{\text{max}} - z_{\text{min}}} \right)^m \text{W}\,\text{m}^{-2}$$

(m3): $m = 3$, (m4): $m = 4$ (reference), (m5): $m = 5$

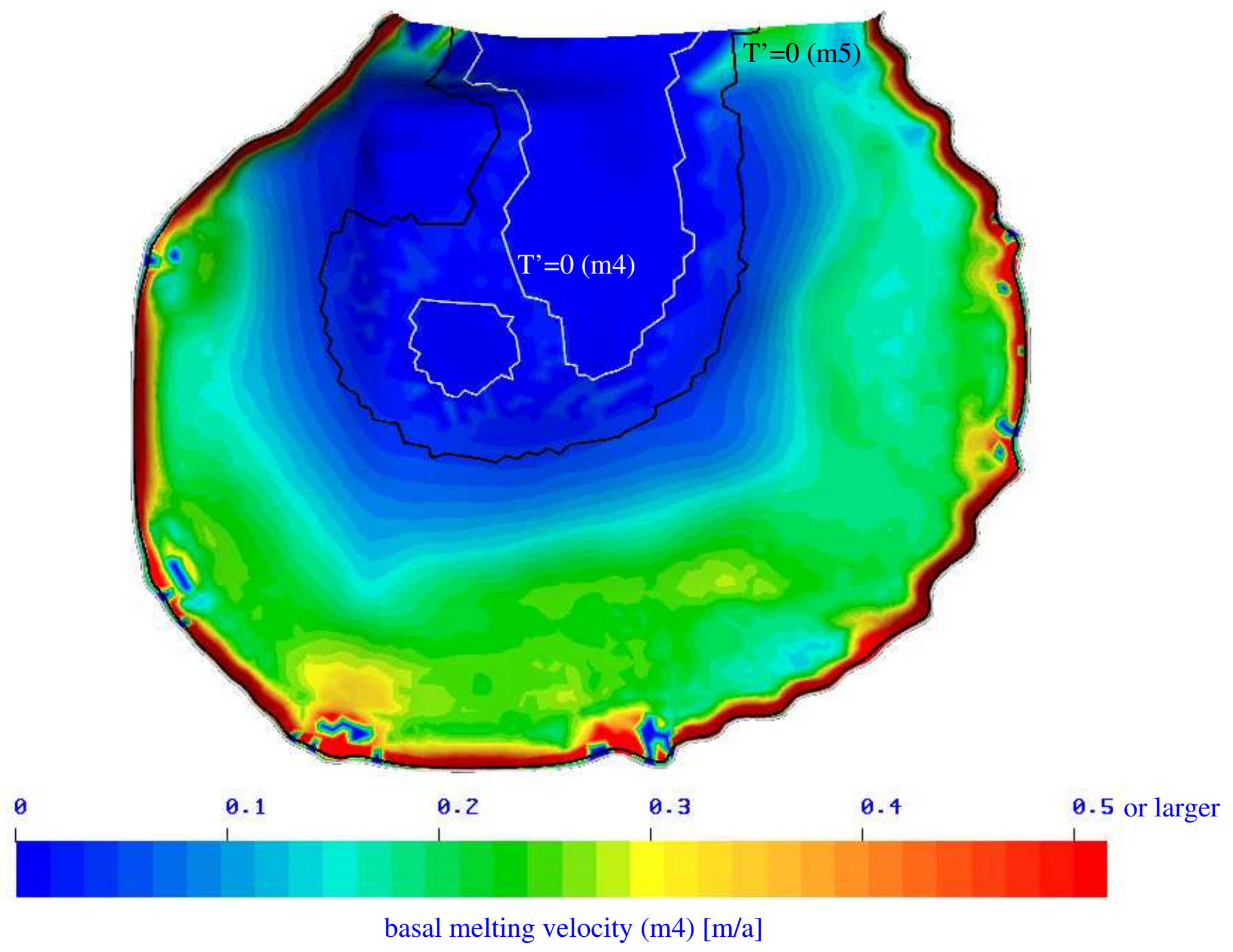
Variation of firn/ice rheology

(m4) reference, compressible firn: $a(D)$, $b(D)$, $E = 1/3$
(a) incompressible firn: $b \rightarrow b = 0$
(b) porous ice ($a \rightarrow a = 1$, $b \rightarrow b = 0$, $E \rightarrow E/D$)
(c) pure ice ($a \rightarrow a = 1$, $b \rightarrow b = 0$, $E \rightarrow E = 1$)

Results



⇑ Temperature, T, age, $\mathcal{A}$, and velocity, v_y, v_z, profiles over depth, d, at the K2 bore-hole position.



⇐ Basal melting velocity (m4) and levels for $T' = 0$ (m4,m5); cases (m3), (a), (b), (c) all show completely temperate base

Discussion and outlook

Firn compressibility has to be taken into account. Else, there is too less downward convection of cold/young firn from the free surface, leading to temperate/too old ice in the lower parts of the glacier. Suggested future work:

- introduction of a mass balance for D
- extension of computational domain beyond outflow

References:

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[3] Franca, L.P., Frey, S.L., Hughes, T.J.R., Stabilized finite element methods: II The incompr. Navier-Stokes equations *Comp. Meths. Appl. Mech.*, 95, 253–276, 1992
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